Lect. 2: Light as EM Waves (Chap. 8 in Cheng)

In the beginning, God said

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \bullet \overline{D} = \rho \qquad \overline{D} = \varepsilon \overline{E}$$

$$\nabla \bullet \overline{B} = 0 \qquad \overline{B} = \mu \overline{H}$$

Then, there was light!

➔ Maxwell's Equations

Material Parameters $\begin{cases} \varepsilon : \textit{permittivity} \\ \mu : \textit{permeability} \end{cases}$

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \nabla^2 \overline{H} = \mu\varepsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

Wave Equations

(source free, uniform medium)

➔ Governs light propagation

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Solutions for Wave Equations

$$\nabla^2 \overline{\mathrm{E}} = \mu\varepsilon \frac{\partial^2 \overline{\mathrm{E}}}{\partial t^2}$$

Assume a plane-wave solution. For example, $\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$

$$\nabla^2 \overline{E} = \overline{x}(-k^2) E_0 e^{j(\omega t - kz)}$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

$$\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \overline{x}\mu\varepsilon(-\omega^2) E_0 e^{j(\omega t - kz)}$$

$$f \cdot \lambda = \frac{1}{\sqrt{\mu\varepsilon}}$$

Speed of light!

$$k^2 = \mu\varepsilon\omega^2$$

$$\therefore k = \omega\sqrt{\mu\varepsilon}$$

Why use plane-wave solutions?

(exponential solutions, time-harmonic solutions, phasor notation, …)

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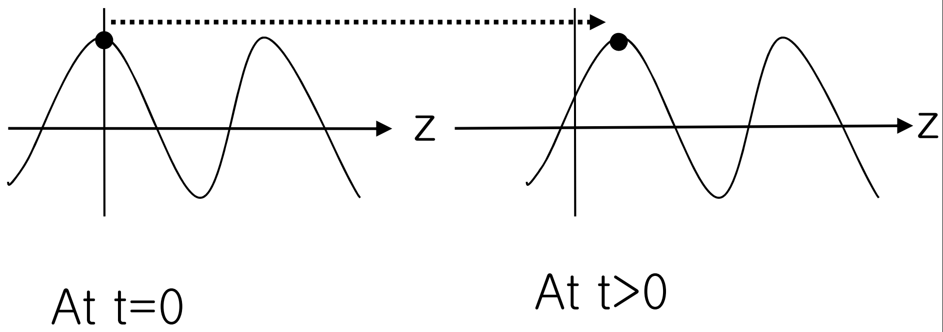
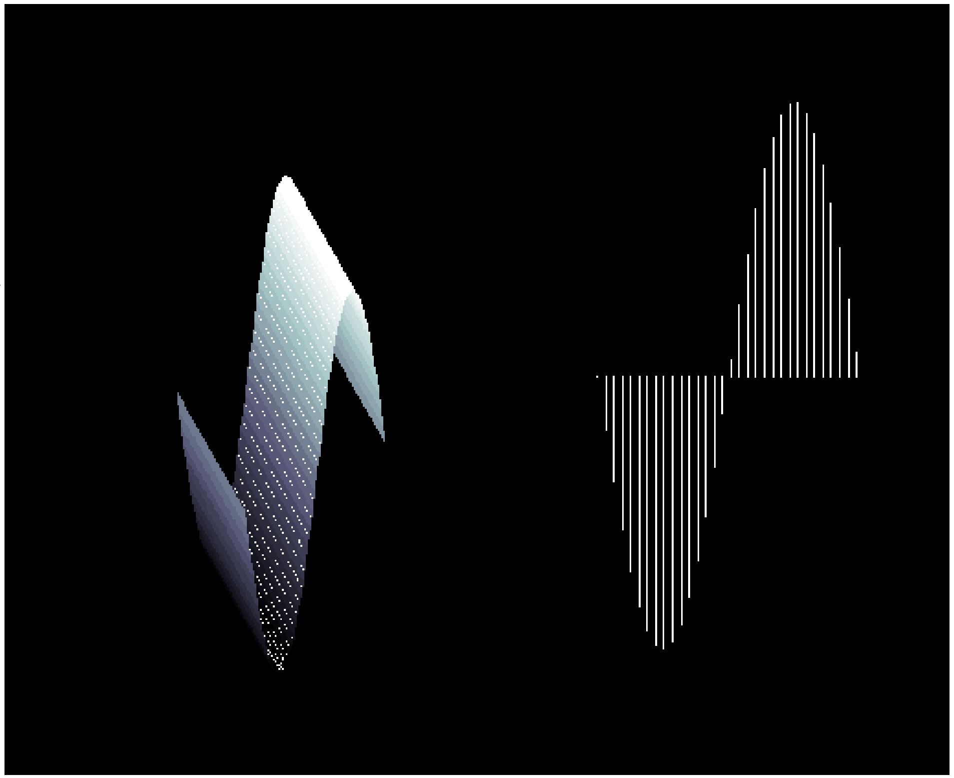
How does the plane-wave solution look like?

For physical representation, $\mathrm{Re}\left[\bar{x}E_0 e^{j(\omega t - kz)}\right] = \bar{x}E_0 \cos(\omega t - kz)$





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How about H-field?

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$$

It can be shown from Maxwell's Equations,

$$\overline{H} = \overline{y} H_0 e^{j(\omega t - kz)}$$

Direction of E, H fields?

Direction of propagation?

Speed of propagation?

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \; [\Omega]$$

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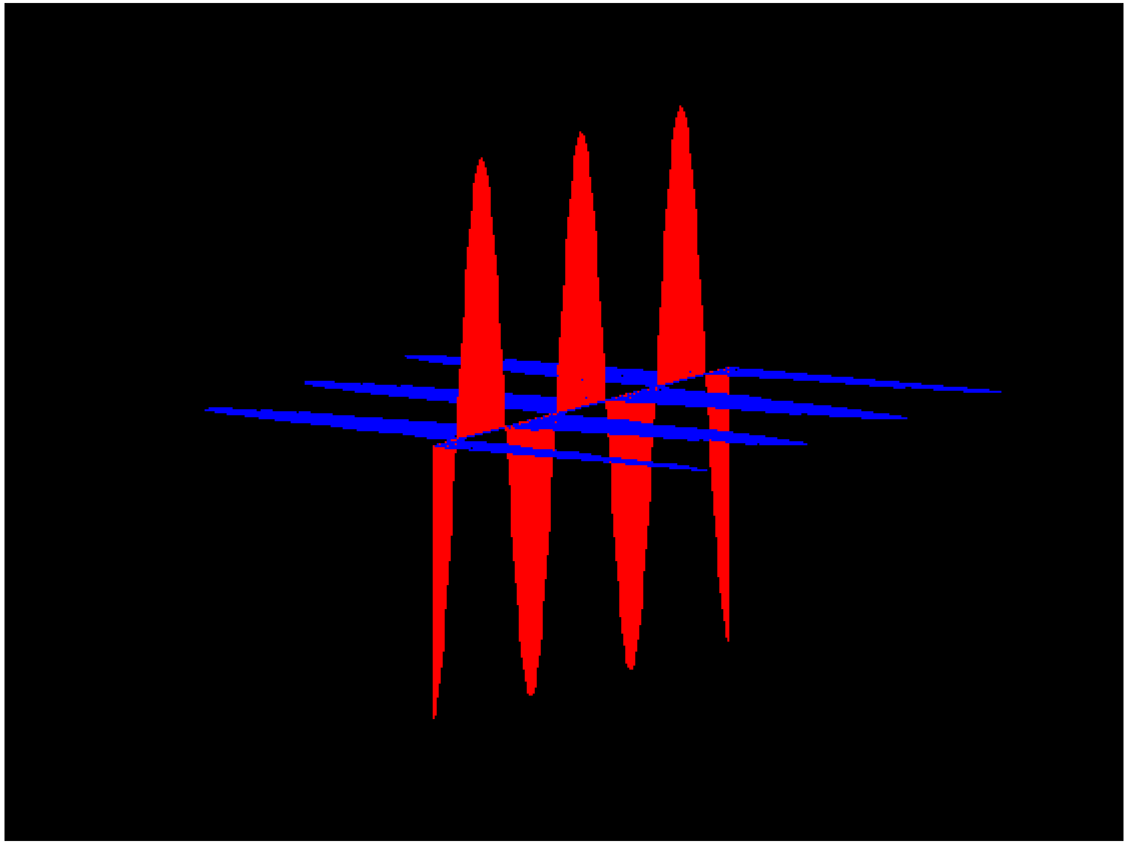
How does the plane-wave solution look like?



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When a wave is propagating into | Plane wave solutions

+z direction: $e^{j(\omega t - kz)}$

-z direction: $e^{j(\omega t + kz)}$

+y direction: $e^{j(\omega t - ky)}$

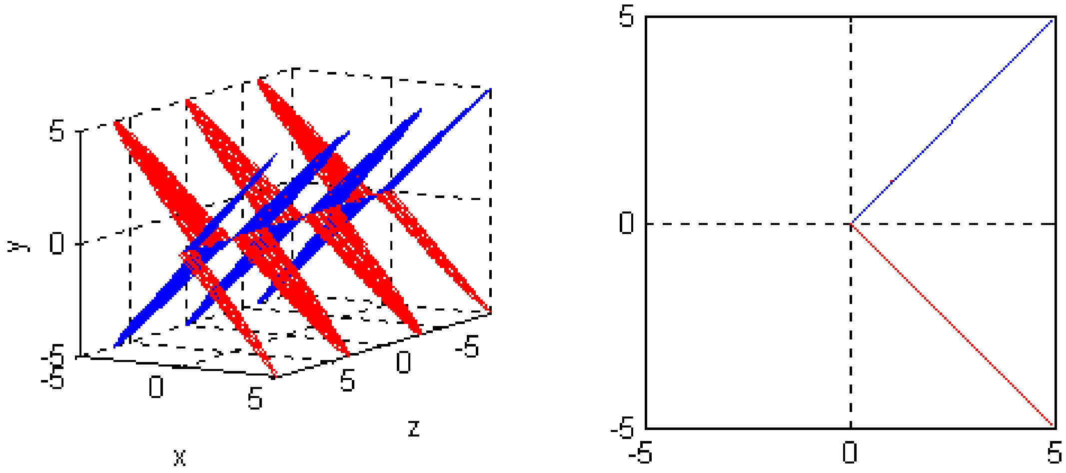
Any direction? $e^{j\omega t} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = e^{j(\omega t - \bar{k} \bullet \bar{R})}$

$$\bar{k} = \bar{x} k_x + \bar{y} k_y + \bar{z} k_z \quad \bar{R} = \bar{x} x + \bar{y} y + \bar{z} z$$

$$\left|\bar{k}\right| = \frac{2\pi}{\lambda}, \ \angle \bar{k}: \text{ direction of propagation}$$

Polarization: Change of E-field direction with time

Linear Polarization $\overline{E} = \left(\overline{x}E_0 + \overline{y}E_0\right)e^{j\omega t}e^{jkz}$



E_x and E_y in-phase

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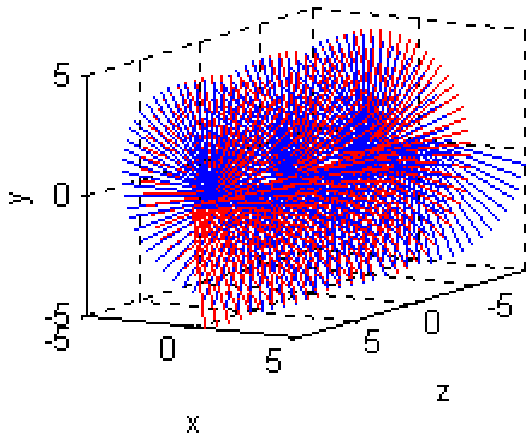
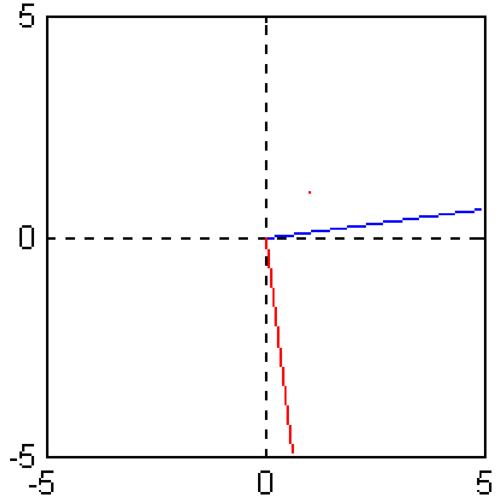
Circular Polarization

$$\overline{E} = \left(\overline{x}E_0 + \overline{y}\,jE_0\right)e^{j\omega t}e^{jkz}$$





E_x and E_y out-of-phase   Handedness?

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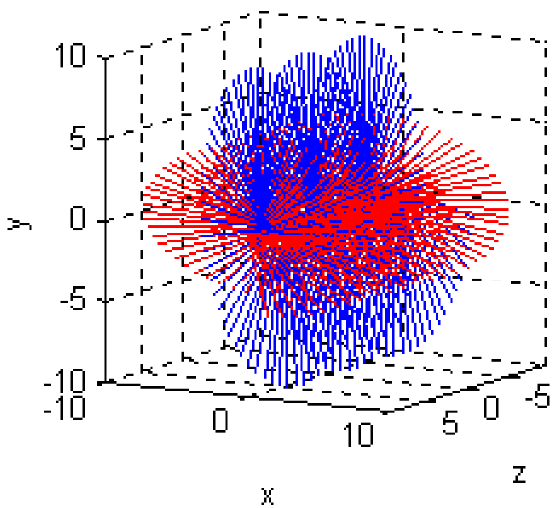
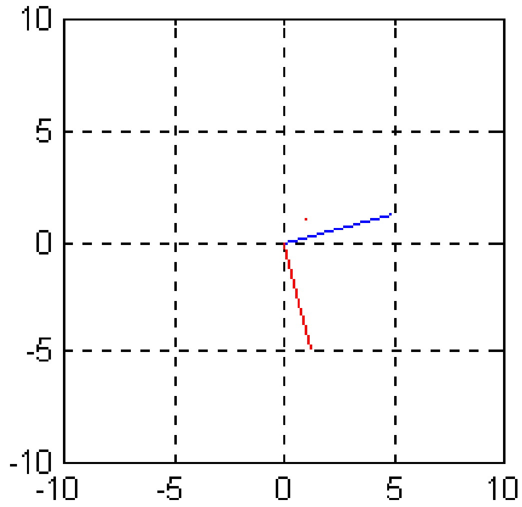
Elliptical Polarization

$$\overline{E} = \left(\overline{x}E_0 + \overline{y}j2E_0\right)e^{j\omega t}e^{jkz}$$





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Homework: Due on 9/13 before Tutorial

A uniform plane wave propagating in a dielectric medium has the E-field given as

$$\overline{E}(t,z) = \overline{x}\,2\cos(10^8 t - z/\sqrt{3}) + \overline{y}\sin(10^8 t - z/\sqrt{3})$$

(a) What is the dielectric constant of the medium?

(b) What type of polarization does the wave have?

(c) What is the corresponding H-field?